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VIBRATIONAL ENERGY ANALYSIS OF GROUND/ STRUCTURE INTERACTION IN TERMS OF WAVE TYPE

M. VILLOT AND J. CHANUT

C.S.T.B. (*Centre Scientifique et Technique du Bâtiment*), 24, *Rue Joseph Fourier*, 38400 *Saint-Martin-d'Hères*, *France*

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The vibrational interaction of underground structures such as tunnels or building foundations with the ground involves different types of waves both in structures and in ground. In this paper, a wave approach is used to study these waves in the simple case of an infinite slab laying on a half-space homogeneous soil. The results are expressed in terms of energy quantities such as input line mobility and radiated power in the case of line force excitation of the slab and power injected to the slab in the case of forced vibration by an incoming ground borne wave. It is shown that in the case of line force excitation of the slab, the radiation loss factors obtained are 10 times higher than common values of structural loss factors, showing that the energy losses of an underground structure are mainly radiated into the ground. In the case of forced vibration by an incoming ground-borne wave, the slab wave type depends on the type of incident ground-borne waves: P waves mainly generate quasi-longitudinal waves in the slab at low frequencies and bending waves at higher frequencies; SV waves generate both bending and quasi-longitudinal waves with a percentage depending on the type of ground.

0. INTRODUCTION

The vibrational interaction of underground structures such as tunnels or building foundations with ground involves different types of wave both in structures and in ground. In this paper, a wave approach is used to study these waves. Only the simple case of an infinite slab lying on a half-space homogeneous soil is considered in a 2-D configuration in order to have a good understanding of the physical mechanisms involved (Figure 1). Both the case of a slab directly excited by a line force and radiating into the surrounding ground and the case of forced vibration when a slab is excited by an incoming ground-borne plane wave are considered.

A more detailed presentation of this vibrational ground/structure interaction problem is given in references [1, 2], including the derivation of the wave approach applied to a slab lying on the ground and also including numerical calculations in the case of horizontal line force excitation and out-of-plane excitation (transverse line force or incoming ground-borne shear SH waves).

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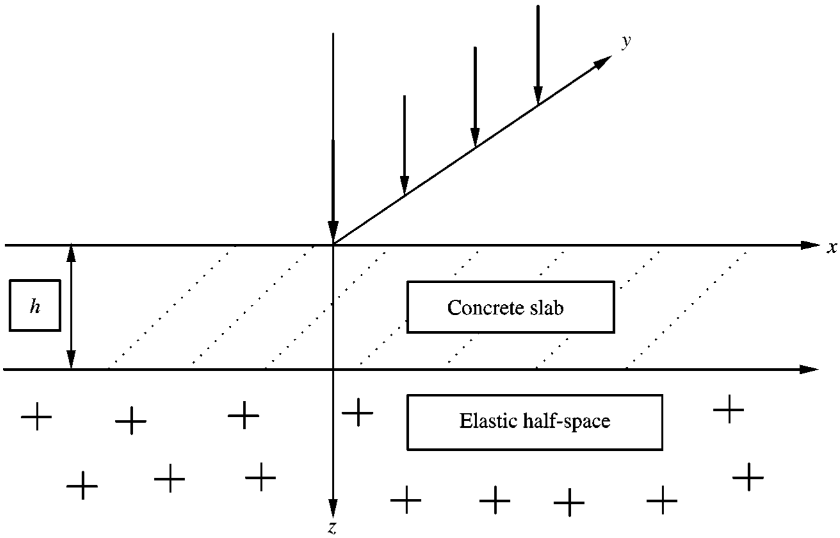

Figure 1. Configuration studied (line force excitation case).

1. THE WAVE APPROACH

The vibrational fields in the slab (considered as a thick solid layer) and in the half-space ground are decomposed into plane compressional and shear waves of wave number k_p and k_{sv} respectively. The equations connecting normal stresses, shear stresses, and normal velocities and tangential velocities at the different interfaces are written and solved in the wave number domain with respect to the x direction (K_X space). Quantities in the real space can then, if needed, be calculated using a 1-D inverse spatial Fourier transform. The corresponding derivation can be found in references [1, 2].

2. ENERGY QUANTITIES CONSIDERED

2.1. PROPAGATION CONSTANTS IN THE SLAB

Using a wave approach, the normal and tangential velocities $\tilde{V}_Z$ and $\tilde{V}_X$ of the slab are readily obtained in the K_X space as wave number spectra. For each spectrum, the wave numbers of the maxima correspond to free waves in the slab and can therefore be identified as the slab propagation constants. Furthermore, the wave number spectra obtained for every frequency can be gathered in a 3-D view (wave number/frequency/velocity component magnitude) in which the dispersion curve of these waves can be identified (see Section 3 for numerical results).

2.2. THE CASE OF VERTICAL LINE FORCE EXCITATION

2.2.1. *Slab input mobility*

The complex input mobility $M(\omega)$ corresponding to a line force $F(\omega)$ normal to the slab can be given by

$$M(\omega) = V_Z(x = 0, \omega)/F(\omega), \tag{1}$$

where $V_Z(x = 0, \omega)$ is the normal velocity in the real space at the excitation line; for a given frequency, $V_Z(x = 0, \omega)$ is calculated through an inverse Fourier transform of the K_X spectrum $\tilde{V}_Z$.

This mobility can be used to calculate the power injected to the slab:

$$P_{inj}(\omega) = \tfrac{1}{2}\,\mathrm{Real}(M(\omega)).\,|F(\omega)|^2 \tag{2}$$

2.2.2. *Radiated power*

The power P_R radiated into the surrounding medium can be calculated in the wave number domain (according to Parseval's theorem) from the z component of the structural intensity I_Z normal to the interface slab/soil:

$$P_R = \int_X I_Z(x)\,\mathrm{d}x = \int_{k_x} \tilde{I}_Z(k_X)\,\mathrm{d}k_X. \tag{3}$$

Expressing $\tilde{I}_Z$ in terms of stress and velocity leads to

$$P_R = \tfrac{1}{2}\,\mathrm{Re}\left[\int_{k_x} (\tilde{\sigma}_z \tilde{V}_z^* + \tilde{\tau}_{xz} \tilde{V}_x^*)\,\mathrm{d}k_x\right], \tag{4}$$

where * denotes a conjugate complex value.

A radiation loss factor η_R can be evaluated from the power radiated and the vibrational bending energy stored in the plate as follows:

$$\omega \eta_R m S \langle V_Z^2 \rangle = P_R, \tag{5}$$

where m is the mass per unit area of the plate and S its infinite area; $\langle\ \rangle$ denotes a spatial average. The product $S\,\langle V_Z^2\rangle$ represents the spatial integral of V_Z^2 which can be evaluated in the wave number domain (according to Parseval's theorem):

$$S\langle V_Z^2\rangle = \int_{k_x} |\tilde{V}_Z|^2\,\mathrm{d}k_X. \tag{6}$$

2.3. THE CASE OF GROUND BORNE WAVE EXCITATION

The incident energy of an incoming P or SV plane wave at incidence θ can be evaluated in the wave number domain from plane wave intensities:

$$\begin{aligned} \tilde{I}_{P,inc}(\omega,\theta) &= \tfrac{1}{2}\rho c_P |\tilde{V}_P|^2 \cos(\theta), \\ \tilde{I}_{SV,inc}(\omega,\theta) &= \tfrac{1}{2}\rho c_{SV} |\tilde{V}_{SV}|^2 \cos(\theta) \end{aligned} \tag{7}$$

and the corresponding net energy transmitted to the slab through the ground/slab interface can be calculated as

$$\tilde{I}_Z(\omega, \theta) = \tfrac{1}{2}\,\mathrm{Re}(\tilde{\sigma}_Z \tilde{V}_Z^* + \tilde{\tau}_{XZ} \tilde{V}_X^*). \tag{8}$$

Note that the wave number k_X is directly related to the angle of incidence $\theta: k_X = k_P \,.\, \sin\theta$ or $k_{SV} \,.\, \sin\theta$.

In the case of an incident diffuse vibrational field, equations (8) and (9) can be integrated as follows:

$$I_P(\omega) = \tfrac{1}{2}\,\rho c_P \int_0^{\pi} |\tilde{V}_P|^2 \cos(\theta)\,\mathrm{d}\theta,$$

$$I_{SV}(\omega) = \tfrac{1}{2}\,\rho c_{SV} \int_0^{\pi} |\tilde{V}_{SV}|^2 \cos(\theta)\,\mathrm{d}\theta, \tag{9}$$

$$I_Z(\omega) = \tfrac{1}{2}\,\mathrm{Re} \int_0^{\pi} (\tilde{\sigma}_Z \tilde{V}_Z^* + \tilde{\tau}_{XZ} \tilde{V}_X^*)\,\mathrm{d}\theta.$$

3. NUMERICAL APPLICATION

The configuration of a concrete slab laying on three different soils is studied; Table 1 contains the material properties of the slab and the soils considered. The behaviour of the slab is analyzed in a 0–200 Hz frequency band which is the usual band of interest for ground-borne vibration from railways. A 0·6 m thick slab is considered for line force excitation and a 0·3 thick slab (more realistic for building foundations) for incoming ground-borne waves.

3.1. THE CASE OF VERTICAL LINE FORCE EXCITATION

3.1.1. *Propagation constants in the slab*

Only the example of a slab on loess (see Table 1) is presented here. Figure 2 shows the 3-D views (wave number/frequency/vertical velocity magnitude $\tilde{V}_Z$) obtained with a vertical line force excitation; mainly bending waves are excited in the plate

TABLE 1

Materials properties of the plate and the soils

	Concrete	Sand	Loess	Hard
Young's modulus E (M Pa)	25×10^3	46	269	1076
Density ρ (kg/m^3)	2500	1700	1550	1550
The Poisson ratio ν	0·15	0·26	0·26	0·26
Loss factor η	0·01	0·08	0·01	0·01

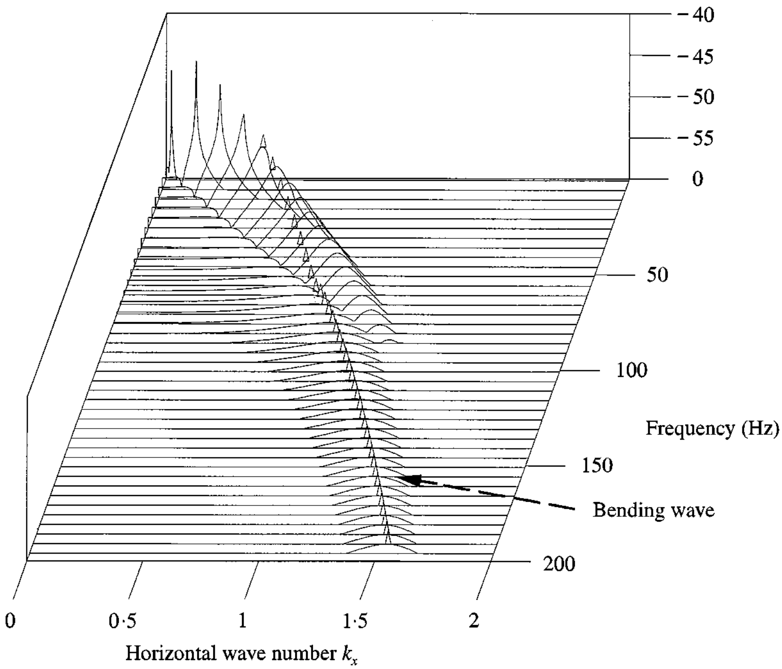

Figure 2. Modulus of the vertical velocity $\tilde{V}_Z$ (dB).

and one crest with a discontinuity around 80 Hz shows some discrepancies from the *in vacuo* bending wave dispersion curve (shown as triangles). The presence of quasi-longitudinal waves in the plate would appear in the 3-D map of the horizontal velocity magnitude $\tilde{V}_X$ (not shown here), but at levels much lower. With vertical excitation, for all practical purposes, only bending waves are generated in the plate.

3.1.2. *Input mobilities*

Figure 3 presents the real part of the input mobility (directly related to the power injected according to equation (2)); the input mobility of the *in vacuo* plate, calculated using the same wave number method, is also plotted (dashed line). At high frequencies (100 Hz and above), the mobilities (as well as the power injected) are practically un-modified by the soil. At low frequencies, the mobilities are lower (compared to the *in vacuo* plate) and depend strongly on the type of ground: the harder the ground, the lower the mobility (and the power injected).

3.1.3. *Radiated power*

The radiation loss factor can be estimated from equations (5), (4) and (6). The radiation loss factors obtained (Figure 4) are about 10 dB higher than common values of total loss factor of *in vacuo* plates, which includes both internal energy loss

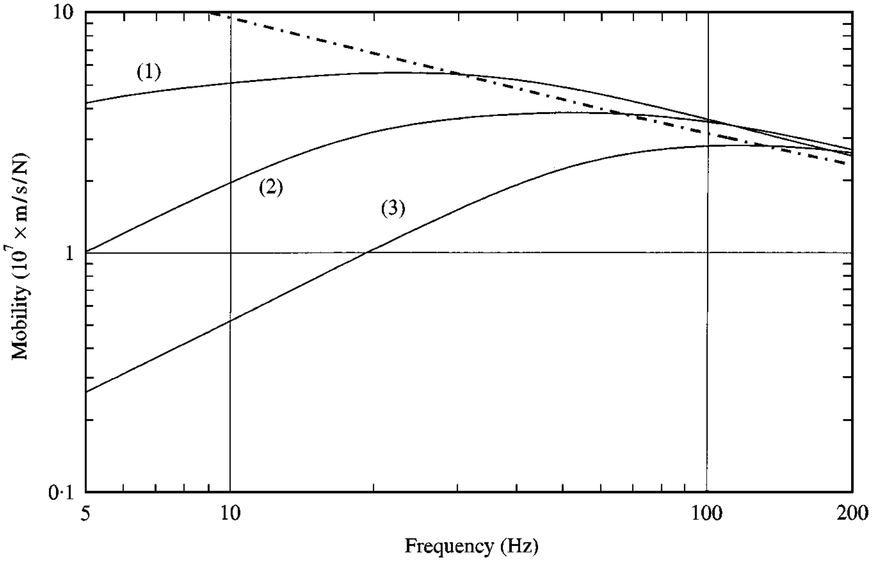

Figure 3. Real part of the input mobility. Soil # (1): Sand. (2): Loess. (3): Hard.

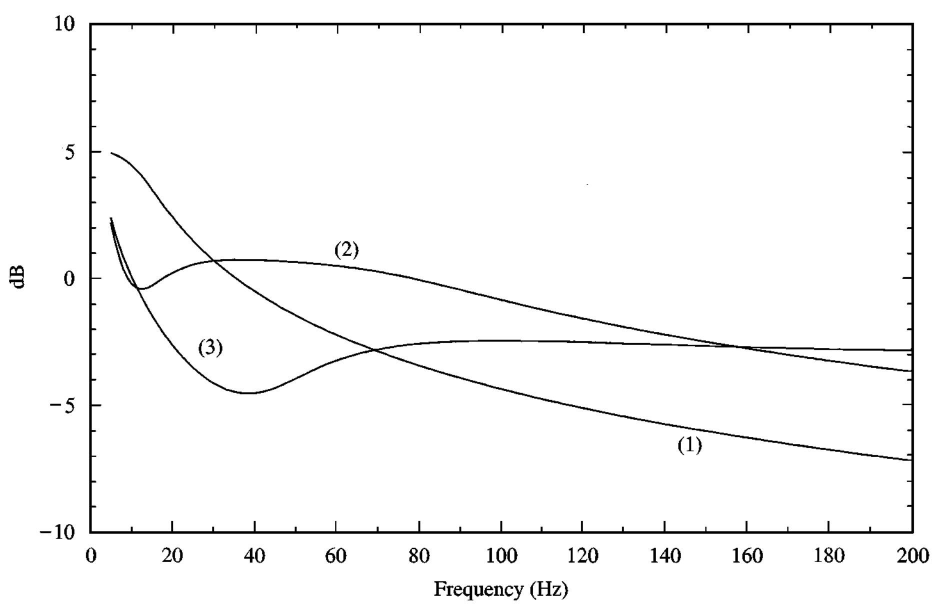

Figure 4. Radiation loss factor. Soil # (1): Sand. (2): Loess. (3): Hard.

and energy flow towards other parts of the structure. Therefore, the energy loss is mainly radiated into ground and the radiated power is practically equal to the injected power; the shape of the radiated power frequency spectra is therefore very close to the shape of the mobility spectra given in Figure 3.

3.2. THE CASE OF GROUND-BORNE INCOMING WAVES

3.2.1. *Identification of the slab wave type*

Using equation (8), the net energy transmitted to the slab can be calculated in the K_X space for each frequency. The corresponding 3-D views (wave number/frequency/intensity magnitude $\tilde{I}_Z$) obtained for P and SV incoming waves are shown in Figures 5 and 6 respectively. The modified dispersion curves corresponding to quasi-longitudinal and bending waves in the slab are also plotted (triangles).

For incident P waves, for all practical purposes only quasi-longitudinal waves are generated in the low-frequency range (below 100 Hz) and only bending waves are generated above. The dispersion curve for quasi-longitudinal waves is substantially modified at low frequencies, tending towards the soil compressional curve at grazing incidences.

For incident SV waves, both quasi-longitudinal and bending waves are generated in the slab, the former mainly at small angles of incidence and the latter at angles close to 90°. An exception can be seen at very low frequency (below 20 Hz) where quasi-longitudinal waves are dominant.

3.2.2. *Diffuse field excitation*

The net energy transmitted to the slab is now integrated over all the incidences to obtain diffuse field energy frequency spectra; the incident wave amplitudes are chosen so that the diffuse field incident energies I_P and I_{SV} of equations (9) are the same. The spectra obtained for both incident P and SV waves are given in Figure 7 for the three types of soil already mentioned.

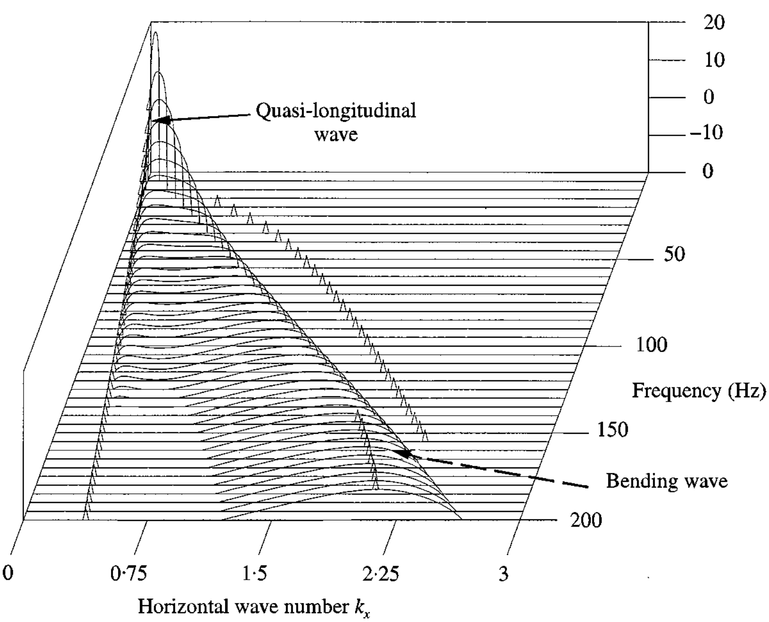

Figure 5. Normal intensity $\tilde{I}_Z$ (dB); incident P wave.

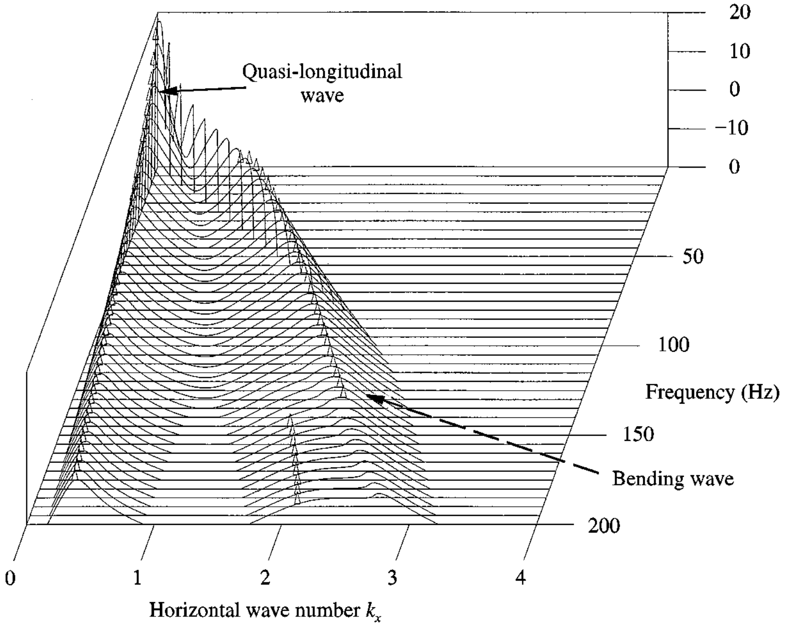

Figure 6. Normal intensity $\tilde{I}_Z$ (dB); incident SV wave.

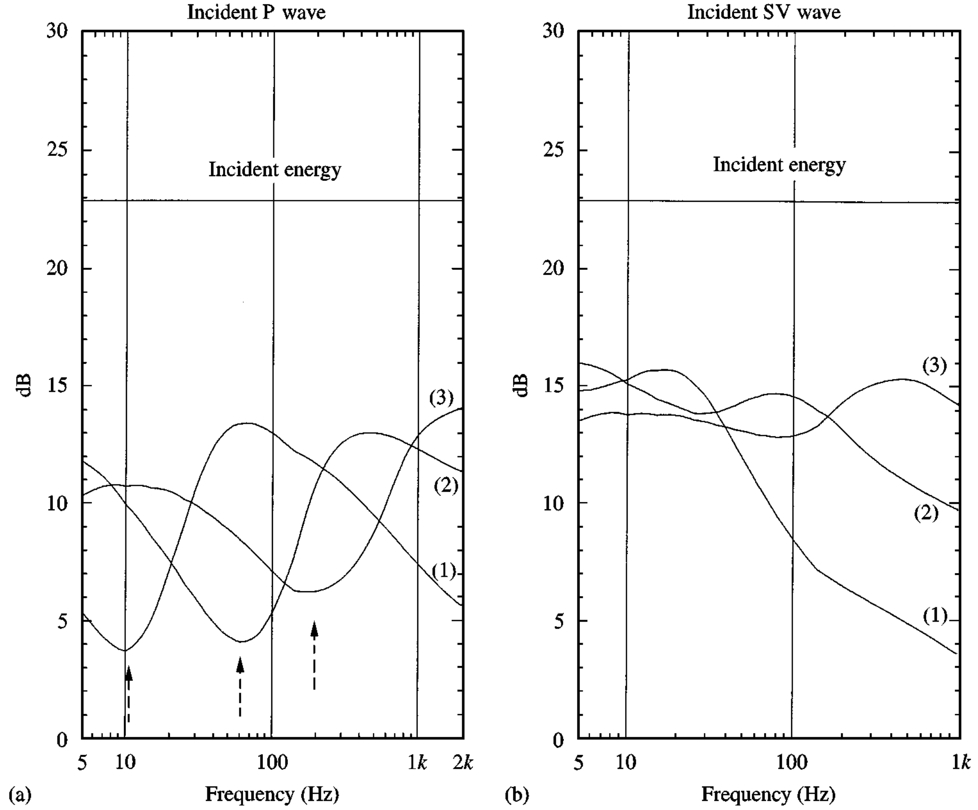

Figure 7. Diffuse field injected energy. Soil # (1): Sand. (2): Loess. (3): Hard.

For incident P waves (Figure 7(a)), the spectra obtained are not very smooth. For each type of soil, a low transmitted energy region (indicated by an arrow) separates quasi-longitudinal waves (below) and bending waves (above) generated in the slab. For incident SV waves (Figure 7b), smoother spectra are obtained.

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